APPROXIMATE $\sin 1$ USING $T_5(x)$ AND FIND AN INTERVAL FOR $\sin 1$.

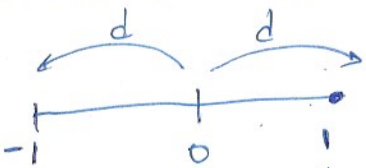
$f(x) = \sin x$ $\qquad$ $\sin 1 \approx T_5(1) = 1 - \frac{1}{3!} + \frac{1}{5!} = 1 - \frac{1}{6} + \frac{1}{120} = \frac{120-20+1}{120}$

$$T_5(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \qquad\qquad = \frac{101}{120} = 0.841\overline{6}$$

NEED TO ESTIMATE $R_5(1)$

① $|R_5(x)| \le \frac{M}{6!}|x-0|^6$ $\qquad$ WHERE $|f^{(6)}(x)| \le M$

FOR $|x-0| \le d = 1$



WE WANT TO LET $x=1$ IN T.S.

$|f^{(6)}(x)| = |-\sin x| \le \frac{\sqrt{3}}{2}$

FOR $|x-0| \le 1$

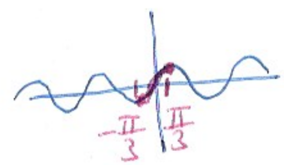
USE $M = \frac{\sqrt{3}}{2}$

$f^{(6)}(x) = -\sin x$

$|-\sin x| \le M$ ON $[-1,1]$

$$|R_5(x)| \le \frac{\frac{\sqrt{3}}{2}}{6!} x^6$$

$$|R_5(1)| \le \frac{\frac{\sqrt{3}}{2}}{6!} 1^6 = \frac{\sqrt{3}}{2 \cdot 6!}$$



$\frac{\pi}{3} \approx 1.05$

SO $\frac{101}{120} - \frac{\sqrt{3}}{2 \cdot 6!} < \sin 1 < \frac{101}{120} + \frac{\sqrt{3}}{2 \cdot 6!}$

ON $[-\frac{\pi}{3}, \frac{\pi}{3}]$

$|\sin x| \le \frac{\sqrt{3}}{2}$

$0.8404638536 < \sin 1 < 0.8428694797$

ALSO TRUE ON $[-1, 1]$

(WHICH IS INSIDE $[-\frac{\pi}{3}, \frac{\pi}{3}]$)

IF YOU WANT TO APPROXIMATE $\sin 1$ WITHIN 10^{-7}

$$|R_n(x)| \leq 10^{-7}$$

$$|R_n(x)| \leq \frac{M}{(n+1)!} x^{n+1} \quad \text{ON } [-1,1]$$

SO IF WE MAKE $\left| \frac{M}{(n+1)!} x^{n+1} \right| \leq 10^{-7}$ ON $[-1,1]$

$|x^{n+1}| \leq 1$ ON $[-1,1]$

$$\frac{\frac{\sqrt{3}}{2}}{(n+1)!} \leq 10^{-7}$$

$$\underbrace{10^{7} \cdot \frac{\sqrt{3}}{2}}_{\cancel{11,547,005}\ 8,660,254} \leq (n+1)!$$

$$\boxed{n=10}$$

NEED $T_{10}(1) = \boxed{T_9(1)}$

$$T_9(1) = 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!}$$

OR TO ESTIMATE $R_5(1)$

(2) ALTERNATING SERIES, SO

$$|R_n(x)| < |a_{n+1}|$$ REMOVED SIGNS OF ALT SERIES

a_{n+1}

$$\sin x \approx T(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

$n=1$ 2 3 4 5

$$a_n = \frac{(-1)^{n+1} x^{2n-1}}{(2n-1)!}$$

$$a_{n+1} = \frac{(-1)^{n+2} x^{2(n+1)-1}}{(2(n+1)-1)!} = \frac{(-1)^{n+2} x^{2n+1}}{(2n+1)!}$$

IF WE WANT $|R_n(1)| \le 10^{-7}$

$$\left| \frac{\cancel{(-1)^{n+2}} \cancel{x}^{2n+1}}{(2n+1)!} \right| \le 10^{-7} \longrightarrow \frac{1}{(2n+1)!} \le 10^{-7}$$

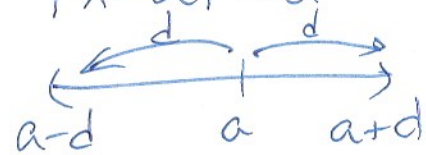
$$10^7 \le (2n+1)!$$

$n \ge 5 \rightarrow$ 5 TERMS OF ALT SERIES

$$1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!}$$

SHORTCUT TO FINDING M

$$|f^{(n+1)}(x)| \leq M \text{ for } |x-a| \leq d$$



eg. $f^{(n+1)}(x) = (x^3-3x^2+2x-7)e^{2x}$ for $|x+1| \leq 2$

$$-1-2 \leq x \leq 1+2$$

$$-3 \leq x \leq 1$$

$$|(x^3-3x^2+2x-7)e^{2x}|$$

$$\leq (|x^3| + |-3x^2| + |2x| + |-7|)|e^{2x}|$$

$$\leq (|(-3)^3| + |-3(-3)^2| + |2(-3)| + 7)e^{2(1)}$$

$$= (27+27+6+7)e^2$$

$$= 67e^2 \leftarrow M$$

$$e^x \approx T(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$ $\leftarrow$ INTERVAL OF CONV IS $(-\infty, \infty)$

HOW WE KNOW THE SERIES CONV TO e^x FOR ALL x?

$$|R_n(x)| \le \frac{M}{(n+1)!} x^{n+1}$$ WHERE $\left|(e^x)^{(n+1)}\right| \le M$

$$|R_n(x)| \le \left|\frac{e^d}{(n+1)!} x^{n+1}\right| \le \frac{e^d \cdot |d|^{n+1}}{(n+1)!}$$

FOR $-d \le x \le d$.

$e^x \le M$ FOR $|x-0| \le d$.

$e^x \le e^d$ FOR $-d \le x \le d$

USE $M = e^d$

AS $n \to \infty$

IE. $$\lim_{n\to\infty} \frac{e^d |d|^{n+1}}{(n+1)!} = e^d \lim_{n\to\infty} \frac{|d|^{n+1}}{(n+1)!} = e^d \cdot 0 = 0$$

USE $\lim_{n\to\infty} \frac{x^n}{n!} = 0$ FOR ALL $x \in \mathbb{R}$

$$\frac{|d|^2}{2!}, \quad \frac{|d|^3}{3!}, \quad \frac{|d|^4}{4!}$$

$* \frac{|d|}{3}$ $\quad * \frac{|d|}{4}$ $\quad * \frac{|d|}{5}$

$* \frac{|d|}{\lceil d \rceil} \le 1$ $\quad * \le 1$ $\quad * \le 1$ $\quad * \le 1$

ADDITIONAL FACTORS GO TO 0

CEILING OF d = SMALLEST INTEGER BIGGER THAN OR EQUAL TO d

IE, REGARDLESS OF X,

$$|R_n(x)| \to 0$$

$$|e^x - T_n(x)| \to 0$$

$$e^x - T_n(x) \to 0$$

$$-T_n(x) \to -e^x$$

$$T_n(x) \to e^x$$

$$T(x) = e^x$$